CHAPTER 2

PRIMARY SCHOOL MATHEMATICS IN THE NETHERLANDS

The Perspective of the Curriculum Documents

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In the Netherlands, the school system consists of three stages: primary education; secondary education; and higher education (see Figure 2.1). Primary school is for students in the age range from 4 to 12 years and starts with two kindergarten grades (Grades K1 and K2), which are followed by six primary school grades (Grades 1–6). Secondary education is divided

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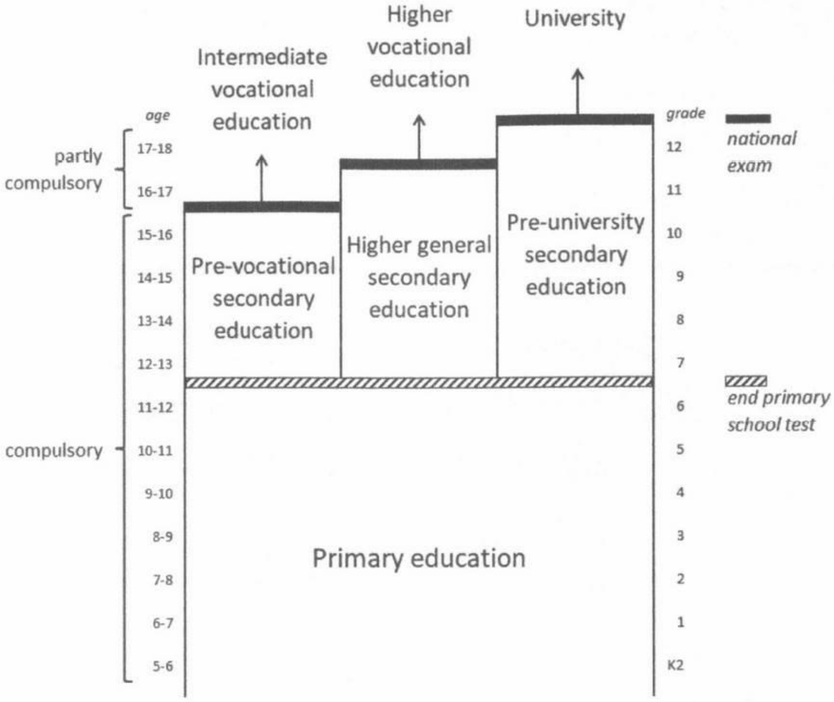

Figure 2.1 The Dutch educational system.

into three different levels with several sub-levels, and for these three levels the number of grades differs. Higher education includes vocational education and university education. Although each level of secondary education is meant to prepare students for a particular form of higher education, it is also possible for students to switch between levels. For example, a student who has attained a diploma in HAVO (higher general secondary education) can then go to the fifth and sixth grade of VWO (pre-university secondary education), and after that can go to university.

Children can go to school when they are 4 years old, but education is compulsory from the age of 5 until 16. After this age, education is partly compulsory, which means that students have to continue school until their 18th birthday or until they acquire a diploma (of HAVO, VWO or intermediate vocational education), whichever comes first.

In this chapter, we discuss the mathematics curriculum for the primary school stage. The reason for this choice is that, in the Netherlands, primary education has a longer history than secondary education in thinking about the goals to be achieved by the students. In primary education, the first goal prescriptions were released in 1993, while for secondary education they came only in 2009 and only for the first years of secondary school. For the remaining years, the curriculum is determined by the topics included in the final secondary school examinations. Moreover, the primary school mathematics curriculum is laid down in various curriculum documents, which makes it interesting to investigate how these documents together form the curriculum.

CURRICULUM DOCUMENTS FOR PRIMARY SCHOOL MATHEMATICS EDUCATION

Mathematics education starts in the kindergarten years with doing playful mathematics-related activities. In the grade years, mathematics is taught systematically in daily lessons for about five hours per week. The mathematical content that is taught in primary school is mainly defined in four types of curriculum documents:

- the legally prescribed standards;
- resources describing teaching-learning trajectories;
- textbooks; and
- assessment materials, especially compulsory tests at the end of primary school.

These documents represent different curriculum levels (e.g., Goodlad, 1979; Thijs & Van den Akker, 2009). The legally prescribed standards can be regarded as the *intended curriculum*, that is, the curriculum that describes the desired learning outcomes at a particular time in students' school career. Following Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002), we consider textbooks as a separate level, the *potentially implemented curriculum*, intermediating between the intended curriculum and the implemented curriculum, which refers to the actual teaching and learning processes taking place in school (see Figure 2.2).

The teaching-learning trajectories are a mediating layer between the intended and the implemented curriculum and, therefore, belong to the potentially implemented curriculum. These trajectories sketch learning pathways through which students can achieve the standards that have been determined for the end of primary school. Although the development of these teaching-learning trajectories was initiated and financed by the Ministry of Education, they do not have a statutory status and, thus, they are not part of the formal intended curriculum. Finally, assessment materials influence the implemented curriculum because in these materials the mathematical knowledge, skills, and insights students are supposed to achieve over the school grades are operationalized.

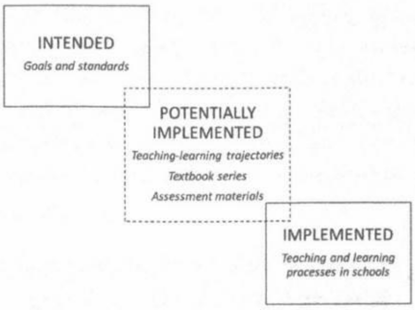

Figure 2.2 Levels of curriculum in the Netherlands (adapted version from Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002).

The aforementioned curriculum documents each have their own role in supporting mathematics education that is realized in primary school and determined by different actors, including the Ministry of Education, SLO (Netherlands Institute for Curriculum Development), CvTE (College for Tests and Examinations), Cito (National Institute for Educational Measurement), textbook authors and publishers, and developers and researchers of the Freudenthal Institute. Our aim with this chapter is to illustrate the primary mathematics curriculum in these documents and to discuss their coherence. However, to understand the role these curriculum documents play in Dutch mathematics education, we first pay attention to the constitutionally established *freedom of education* in the Netherlands.

FREEDOM OF EDUCATION

In the Netherlands, *freedom of education* implies that the government is rather restrained in being involved in how education is realized. The origin of this policy dates to the Dutch Constitution of 1848 that permitted the founding of schools based on a religious denomination (Bakker, Noordman, & Rietveld-van Wingerden, 2010). In 1917, this was followed by a law that regulated that such denominational schools from then on were to receive the same financial resources from the government as public schools (Bakker et al., 2010). A few years later, in 1920, it was decided that this regulation also applied to schools with specific pedagogical approaches (Boekholt & De Booy, 1987).

As a consequence of the restrained policy, before the first Dutch standards could be established in 1993 (Ministry of Education, 1993/1998), eight years of debate occurred around the central question of whether or not governmental prescription of goals was compatible with the freedom of education (Letschert, 1998). Since 2008, after a parliamentary inquiry of educational innovations that had taken place, the government has strived more explicitly than before to make a strict distinction between the *what* (the learning goals and content to be taught) and the *how* (the way in which this content is to be taught) of education. In that year, the parliament stated that the government only prescribes the "what," and not the "how" (Committee Parliamentary Research Education, 2008; Ministry of Education, 2008). In line with this, the government presently sees freedom of education as grounds for the founding of schools based on specific ideas about educational and didactical approaches (Education Council, 2012; Ministry of Education, 2013). Currently, the Ministry of Education (2015) is working on a law amendment for having a renewed interpretation of the freedom of education in this spirit.

As a result of the freedom of education, the Dutch government does not interfere with textbook development and there is no authority that recommends, certifies, or approves textbooks before they are put on the market. This means that there are few restrictions in developing and publishing textbooks. Schools are free to choose a textbook that they think fits most closely to their view on teaching. Regarding the compulsory student test at the end of primary education, schools have limited choice. Schools may use the test that is developed by Cito and commissioned by the government, or may use a test developed by another company but also approved by the Ministry of Education.

THE MATHEMATICS CURRICULUM AS REFLECTED IN STANDARDS

The standards for mathematics education in primary school are described in two ways. The current *Core Goals* document (Ministry of Education, 2006) describes eleven globally formulated goals, which leave much room for interpretation about what mathematics students should learn in primary school. In 2010, the Core Goals document was extended with the *Reference Framework* (Ministry of Education, 2009), which describes in more detail what students should have achieved at the end of primary school (and at the end of secondary education and at the end of intermediate vocational education).

The Core Goals for Mathematics

Figure 2.3 shows the complete list of goals for mathematics as included in the Core Goals document published in 2006. For example, for basic

Mathematical understanding and skills
1. Students learn to use mathematical language.
2. Students learn to solve practical and formal mathematical problems and present their reasoning clearly.
3. Students learn to justify and judge solution strategies for mathematical problems.

Numbers and operations
4. Students learn to understand the structure and interconnectedness of numbers, whole numbers, decimal numbers, fractions, percentages and ratios, and are able to calculate with these in practical situations.
5. Students learn to carry out mentally and quickly the basic operations with whole numbers at least up to 100, whereby the additions and subtractions up to 20 and the multiplication tables are known by heart.
6. Students learn to count and calculate by estimation.
7. Students learn to add, subtract, multiply and divide in clever ways.
8. Students learn written addition, subtraction, multiplication and division in more or less curtailed standardized ways.
9. Students learn to use the calculator with insight.

Measurement and geometry
10. Students learn to solve simple geometry problems.
11. Students learn to measure and calculate with measurement units and measures related to time, money, length, perimeter, area, volume, weight, speed and temperature.

Figure 2.3 The goals in the Core Goals document for mathematics (from Ministry of Education, 2006, pp. 40–45). *Note:* All the quotations and the examples from publications published in Dutch included in this chapter have been translated into English by the authors of this chapter.

number operations, students have to learn to calculate in practical situations, and should be able to calculate mentally and in clever ways, and should be competent to carry out standardized calculation methods in a more or less curtailed way. What "practical situations" include and what these different methods imply is not specified. Regarding the number range, it is only mentioned that mental calculation should at least cover whole numbers to one hundred and that additions and subtractions up to twenty should be known by heart.

In addition to the goals, the Core Goals document also gives a so-called *characteristic of mathematics,* which describes what is valued in mathematics education. Next to the basic mathematical skills and knowledge regarding the relationships and operations that apply to numbers, measurements and structures, more overarching competencies should be valued in mathematics education, such as asking mathematical questions and problem solving. Further, it is emphasized that students should develop mathematical understanding and acquire mathematical literacy. By teachers keeping in mind students' knowledge, competencies, and interests, students "will feel challenged to carry out mathematical activity and that they will be able to do mathematics at their own level, with satisfaction and pleasure" (Ministry of

Education, 2006, p. 39). Students should also learn to respect each other's ways of thinking. Mathematics is, thus, seen as a social activity: in addition to working individually, students have to work in groups and should "learn to use explaining, formulating, notating, and giving and receiving criticism as a specific mathematical method to organize and ground their thinking and to prevent mistakes" (Ministry of Education, 2006, p. 39). A further guideline is that students should learn mathematics in the context of situations that are meaningful to them.

By including these directions in the characteristic of mathematics education, the Core Goals go, in a way, beyond prescribing just the *what* of mathematics education. They also provide a view on the learning of mathematics, which is reflected in the preamble of the Core Goals document. Although it is clearly stated that the given goals do not comment about didactics, which is in line with the freedom of education, the preamble does provide some indications about the ways in which teachers can stimulate students' development, for example, that education should be structured, interactive, and make connections to daily life (Ministry of Education, 2006, pp. 7–9).

The Reference Framework for Mathematics

The Reference Framework was developed as a result of increasing concerns about the mathematical skills of students in secondary and vocational education (Ministry of Education, 2007). This Reference Framework prescribes standards regarding the attainment targets that students should reach at specified points in their schooling, starting from the end of primary school. These attainment targets concern the domains of number, rational numbers and ratios, measurement and geometry, and data handling. For each domain, three competencies are distinguished: using mathematical language, making connections between procedures and concepts, and carrying out applications in contextual situations and bare number problems. Furthermore, for each of these competencies, three performance expectations are formulated: knowing by heart, being able to use, and understanding.

The standards are formulated for three age-related target levels (1S, 2S, 3S), and three minimum levels (1F, 2F, 3F) for students who cannot achieve the S levels. The levels 1S and 1F are meant for the end of primary school and the beginning of secondary education, in which 1S is meant for the majority of students (Expertgroep Doorlopende Leerlijnen, 2008). The 2F, 2S, 3F, and 3S levels are meant for older students. Table 2.1 shows some examples of the intended content and performance expectations for 1F and 1S in the domain of number.

TABLE 2.1 Examples of the Intended Content and Performance Expectations for 1F and 1S in the Domain of Number

Level 1F	Level 1S (Which Also Includes Level 1F)
• Translating a simple problem situation into a number sentence • Rounding off whole numbers to round numbers • Mental calculation: addition, subtraction, multiplication, and division "with zeroes," also with simple decimal numbers: 30 + 50 1200 – 800 65 × 10 3600 ÷ 100 1000 × 2.5 0.25 × 100 • Efficient calculation (+, –, ×, ÷) using the properties of numbers and operations, with simple numbers • Addition and subtraction (including determining the difference) with whole numbers and simple decimal numbers: 235 + 349 1268 – 38 €2.50 + €1.25 • Multiplication of a one-digit number with a two-digit or three-digit number: 7 × 165 5 hours work for €5.75 an hour • Multiplication of a two-digit number with a two-digit number: 35 × 67 • Division of a three-digit number with a two-digit number, with or without a remainder: 132 ÷ 16	• Translating a complicated problem situation into a number sentence • Rounding off decimal numbers to whole numbers • Mental calculation: addition, subtraction, multiplication, and division "with zeroes," also with more difficult numbers, including larger numbers and more complicated fractions and decimal numbers: 18 ÷ 100 1.8 × 1000 • Efficient calculation with larger numbers • Division with a remainder or a (rounded off) decimal number: 122 ÷ 5

Note: From Ministry of Education (2009, pp. 23–26).

As compared with the 1F level, the 1S level generally involves handling more complex problem situations, dealing with more difficult numbers including larger numbers and complicated fractions and decimal numbers, and a higher level of understanding. For example, students have to understand the difference between a digit and a number, the importance of the number zero, and reasoning about questions like: "Does there exist a smallest fraction?" (Ministry of Education, 2009, p. 25).

The way in which the standards in the Reference Framework are formulated is more specific than in the Core Goals document. For example, in the latter document it is just stated that students have to learn to add, subtract, multiply, and divide in clever ways (see Figure 2.3). The Reference Framework is more specific about what these "clever ways" imply, namely that students should learn "efficient calculation using the properties of numbers and operations" (Ministry of Education, 2009, p. 24). In addition, compared to the Core Goals, in the Reference Framework more directions are given regarding the number range. For example, concerning multiplication, students should learn a standard procedure to multiply a three-digit number by a one-digit number, and a two-digit number by a two-digit number. Similar to the Core Goals, the Reference Framework gives no specifications or examples of efficient calculation methods or standard procedures. The same goes for descriptions as *meaningful*, *simple*, and *more complex* context situations. Thus, the Reference Framework, like the Core Goals, leaves much room for interpretation.

The Mathematics Curriculum as Reflected in Teaching-Learning Trajectories

In the years after 1993 when the first Core Goals were published, there was discussion about whether these end-of-primary-school standards were sufficient to ensure that these goals would be achieved (see De Wit, 1997). In particular, there was a plea for having longitudinal teaching-learning trajectories with intermediate attainment targets. In 1997, this plea for such trajectories, which were a new educational phenomenon at that time, was honored. The Ministry of Education commissioned the Freudenthal Institute to develop *TAL teaching-learning trajectories*. The acronym TAL stands for "Tussendoelen annex leerlijnen" [Intermediate attainment targets annex teaching-learning trajectories].

The first TAL trajectory (see Treffers, Van den Heuvel-Panhuizen, & Buys, 1999) was on whole-number arithmetic in the lower grades of primary school and was followed by a trajectory on whole-number arithmetic in the upper grades of primary school (see Van den Heuvel-Panhuizen, Buys, & Treffers, 2001). For the upper grades, a trajectory for rational numbers was also developed (see Van Galen et al., 2005). For the domain of measurement and geometry, a teaching-learning trajectory was developed for both the lower grades (see Van den Heuvel-Panhuizen & Buys, 2004) and for the upper grades of primary school (Gravemeijer et al., 2007).[1] Later, SLO developed online TULE[2] teaching-learning trajectories for all subjects. For mathematics, this TULE document was based on TAL. Because there are only slight differences in content between the TAL and the TULE trajectories, we confine ourselves here to a description of the TAL trajectories and, in particular, to the two on whole-number arithmetic.

In the view of the TAL developers, the term *teaching-learning trajectory*

> ...has three interwoven meanings: a *learning trajectory* that gives a general overview of the learning process of the students; a *teaching trajectory*, consisting of didactical indications that describe how the teaching can most effectively link up with and stimulate the learning process; and a *subject matter outline*, indicating which of the core elements of the mathematics curriculum should be taught. (Van den Heuvel-Panhuizen, 2008, p. 13)

To make the interconnectedness of learning content and didactical approach concrete, in the TAL trajectories on whole-number arithmetic there are intermediate attainment targets to serve as landmarks towards achieving the goals as included in the Core Goals document, together with *teaching frameworks*. These teaching frameworks are descriptions of the teaching-learning processes that are considered to contribute to achieving these targets. For example, regarding addition and subtraction, an intermediate attainment target says that, by the end of Grade 2, students should know how to solve addition and subtraction problems to one hundred, both in context and in a bare number format (see Table 2.2). The corresponding teaching framework indicates that, in order to reach this intermediate attainment target, the teacher should have a good understanding of the nature and function of line and group models to shift students' performance level from applying a counting strategy to a more flexible way of mental calculation and a formal way of operating with numbers.

The intermediate attainment targets and teaching frameworks form the essence of the intended teaching-learning processes. In addition, the TAL trajectories describe in full detail sequences of activities to be done, problems to be solved, strategies to be used, and the models that support these strategies. Thus, TAL provides specifications that are absent in the Core

TABLE 2.2 TAL Intermediate Attainment Target and Teaching Framework for Addition and Subtraction to One Hundred

Addition and Subtraction to 100 at the end of Grade 2	
Intermediate Attainment Target	**Teaching Framework**
By the end of Grade 2, the students have memorized additions and subtractions to ten and have automatized them to twenty. They should then also be able to solve addition and subtraction problems to one hundred, both in context and in a bare number format. The children may use the empty number line, write down intermediate steps, or do it entirely in their heads.	Necessary for the students to reach these attainment targets is that the teacher takes into account the different levels of the students' understanding and adapts the teaching accordingly. The teacher has to have good insight into the nature and function of line and group models. Both models facilitate the transition from the initial calculation by counting to the later, more flexible, formal operation.

Note: From Van den Heuvel-Panhuizen (2008, p. 74), based on Treffers, Van den Heuvel-Panhuizen, & Buys (1999).

Goals and the Reference Framework. For example, for standard calculation methods to one hundred (and beyond), TAL explains both the use of the *stringing strategy* (e.g., calculating 48 + 29 by doing 48 + 20 → 68 + 2 → 70 + 7 → 77) and the *splitting strategy* (e.g., calculating 48 + 29 by doing 40 + 20 = 60 and 8 + 9 = 17 followed by 60 + 17 = 77). Also, for efficient calculation methods, several varying strategies are described, such as *making use of nearby round numbers* (e.g., calculating 48 + 29 by doing 48 + 30 → 78 – 1 → 77) and *raising both terms by 1* (e.g., calculating 77 – 29 by doing 78 – 30). Furthermore, examples are given of the way in which models can be used to support specific calculation methods, such as how an empty number line can be used to solve 48 + 29 by applying a stringing strategy (see Figure 2.4a) and applying a varying strategy (see Figure 2.4b).

Another example of the specifications that TAL provides concerns two forms of written calculation procedures and their interrelatedness for the upper primary grades: whole-number-based calculation and digit-based algorithmic calculation. In the case of a whole-number-based calculation[3] of 463 + 382 (Figure 2.5a), the calculation is carried out with whole-number values working from large to small, that is from left to right (400 + 300 = 700; 60 + 80 = 140; 3 + 2 = 5; followed by 700 + 140 + 5). This calculation can also be carried out in the opposite direction working from small to large, that is from right to left (3 + 2 = 5; 60 + 80 = 140; 400 + 300 = 700; followed by 5 + 140 + 700; Figure 2.5b). By working from right to left, the procedure can be used as an introduction to digit-based algorithmic calculation[4] involving calculating with digits (3 + 2 = 5; 6 + 8 = 14, write down the 4 and carry the 1; 1 + 4 + 3 = 8; Figure 2.5c).

Similar to addition, for subtraction whole-number-based calculation and digit-based algorithmic calculation belong in TAL as common attainment targets for all students. In the case of multiplication, the most curtailed digit-based algorithmic calculation is not considered an attainment target

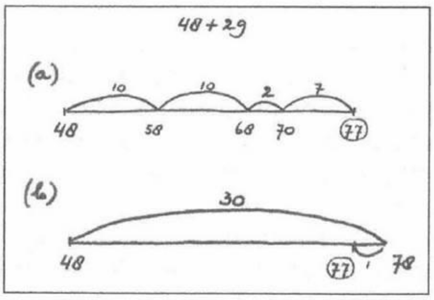

Figure 2.4a and b The use of the empty number line to support different calculation strategies for solving 48 + 29. *Source:* Van den Heuvel-Panhuizen, 2008, p. 67–68; based on Treffers, Van de Heuvel-Panhuizen, & Buys, 1999.

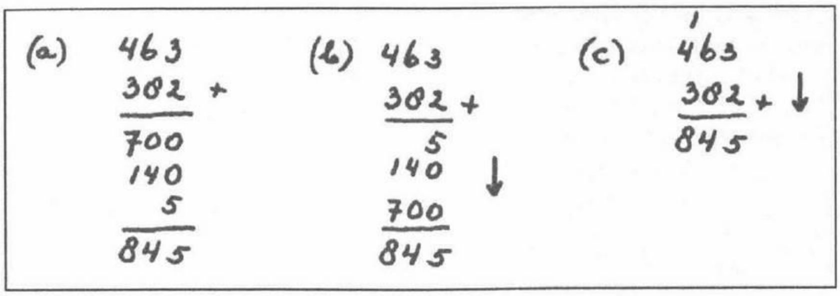

Figure 2.5a–c The addition 463 + 382 by (a) whole-number-based calculation from large to small, (b) by whole-number-based calculation from small to large, and (c) by digit-based algorithmic calculation. *Source:* Van den Heuvel-Panhuizen, 2008, p. 147; based on Treffers, Van den Heuvel-Panhuizen, & Buys, 1999.

for the lesser able students. For division, the traditional long division, the digit-based algorithmic calculation, is not considered to be an attainment target in the TAL trajectory for primary school.

Despite the detailed descriptions of the teaching-learning process for the primary school grades, the TAL trajectories are not meant to offer teachers guidance for their teaching on a day-to-day basis. The main purpose of the TAL trajectories was to bring coherence in primary school mathematics curriculum by providing a longitudinal overview of how children's mathematical understanding develops from K1 and K2 to Grade 6, and how the different stages in this development are connected and are built on each other. An example of this structure is apparent in the three levels that are distinguished in the elementary process of learning to calculate: calculating by counting (e.g., solving number problems by counting on fingers), calculating by structuring (e.g., solving number problems by using the empty number line, see Figure 2.4), and formal calculation (solving number problems by using symbolic notation). The idea is that students can solve problems at different levels, which is also recognizable in the distinction of whole-number-based calculation and digit-based algorithmic calculation. This idea reflects a concentric or spiral approach to teaching, in which a basic foundation is first laid, which later is filled with more complexity and depth. In other words, what is learned in one stage is understood in a later stage at a higher level.

Alongside the domain specific descriptions, TAL explicitly pays attention to the overarching competence of problem solving, emphasizing that students have to work on non-routine problems. For example, for the lower grades of primary school, the problem "Try to make 24 using the following randomly chosen numbers under 10: 3, 4, 7 and 8" is suggested (Van den Heuvel-Panhuizen, 2008, p. 81). In the higher grades, letter problems such

Find the correct digit for each letter.
The problem must match the answer.

```
F O R T Y
    T E N
    T E N +
---------
S I X T Y
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Figure 2.6 Forty and ten and ten is sixty. *Source:* Van den Heuvel-Panhuizen, 2008, p. 167; see also Gardner, 1985, p. 18.

as shown in Figure 2.6, can help students to deepen their understanding of digit-based algorithmic calculation.

THE MATHEMATICS CURRICULUM AS REFLECTED IN TEXTBOOKS

Because a vast majority of Dutch primary school teachers rely heavily in their teaching on the textbook they use (Hop, 2012; Meelissen et al., 2012), mathematics textbook series have a determining role in daily teaching practice (Van Zanten & Van den Heuvel-Panhuizen, 2014). Currently, there are seven mathematics textbook series on the Dutch market, all published by independent, commercial publishers. We focus here on the four most frequently used textbook series as identified by Scheltens, Hemker, & Vermeulen (2013): *De Wereld in Getallen* (WiG; Huitema et al., 2009–2014); *Pluspunt* (PP; Van Beusekom, Fourdraine, & Van Gool, 2009–2013); *Alles Telt* (AT; Van den Bosch-Ploegh et al., 2009–2013); and *Rekenrijk* (RR; Bazen et al., 2009–2013).

All these textbook series provide materials for both students and teachers. Apart from the main books for students, the textbook series also have booklets with additional exercises and software for repetition. For Grades 1 to 6, the textbooks for students are accompanied by extensive teacher guidelines providing detailed information for each daily lesson, including directions for didactical approaches and differentiation. Moreover, these guidelines also provide, for each (sub)domain, grade overviews of the content to be addressed, and the learning goals to be achieved. For the kindergarten years, the textbook series do not have student books but only have source books for the teachers.

All textbook series offer content for numbers and operations (including whole numbers, decimal numbers, fractions, ratios and percentages, and the use of a calculator), measurement (including dealing with length, area, volume, weight, time, speed, temperature, and money), geometry (including activities that can be labeled as orienting, constructing and operating with shapes and figures), and data handling (including dealing with graphs and tables, and calculating the average of values).

Within these (sub)domains, the content and performance expectations included in the textbook series are quite similar. For example, for the domain of numbers and operations, all textbook series contain the automatizing and memorizing of addition and subtraction facts to twenty and the multiplication tables to ten; mental calculation with standard strategies and with varying strategies; estimation; written calculation in one or two standard ways (whole-number-based and digit-based-algorithmic); and making reasoned choices between mental calculation, written calculation, and using a calculator. As an example, Table 2.3 provides an overview of content and performance expectations regarding addition and subtraction in the textbook series WiG.

Although there are many similarities among the four textbook series, there are also differences, mostly related to the sequencing of the content over the grades. For example, for estimation and written calculation, the sequencing differs among the four textbook series (Table 2.4).

The performance expectations are also similar across the four textbook series. For example, they all start the automatization of adding and subtracting to 10 in Grade 1 and to 20 in Grade 2. They all also continue the process of memorizing addition and subtraction facts in Grade 3. Furthermore, all textbook series offer context situations for addition and subtraction from Grade 1 to Grade 6, first with whole numbers and later with decimal numbers in the context of money and bare decimal numbers. Another similarity is that all textbook series provide directions on how to stimulate understanding. An example is that all series explicitly offer ways to encourage students' understanding of place value, for example by using a place value chart and making references to measurement numbers (Figure 2.7).

An example of a difference in performance expectations concerns students' understanding of the relationship between whole-number-based and digit-based written calculation. For example, in WiG, AT, and RR, digit-based algorithmic written multiplication is derived from whole-number-based written multiplication, whereas in PP no relationship is explicitly made between the two forms of written multiplication. Another example concerns written addition and subtraction. RR is the only textbook series that offers whole-number-based addition and subtraction to Grade 6 (Table 2.4), which is related to what this textbook takes as a performance expectation for the lesser able students. In RR, these students may choose to apply a whole-number-based or a digit-based calculation form. Regarding multiplication, WiG and

TABLE 2.3 Overview of Content and Performance Expectations Regarding Addition and Subtraction for Grade 1 to 6 in WiG

Grade	Content and Performance Expectations
1	Addition and subtraction situations are offered for the first time. At the end of Grade 1, students have started with solving addition and subtraction to 20, both in context situations and with bare numbers, and have started automatizing splitting, adding, and subtracting with numbers to 10.
2	Students continue automatizing splitting, adding, and subtracting to 10, and start automatizing addition and subtraction to 20 and later to 100. One of the strategies students learn is making use of analogous problems (4 + 3 → 74 + 3; 8 – 5 → 48 – 5).
3	Students continue automatizing adding and subtracting to 20. Students add and subtract to 1000, by which they make use of the decimal structure of numbers (300 + 40; 560 – 500) and analogous problems to 100 (65 + . . . = 100 → 165 + . . . = 200). All addition and subtraction problems are presented as horizontal number sentences and are calculated mentally in which the use of scrap paper and an empty number line are allowed. A start is made with using clever calculation ways (30 + 30 → 30 + 28) and addition by estimation (205 + 398 ≈).
4	Students add and subtract to 1000 by mental calculation, also in clever ways and by estimation. Hereafter, this is extended to numbers to 10,000 and 100,000, in which students split the numbers, for example, in so many thousands, hundreds, tens, and ones. Students learn whole-number-based written addition and subtraction; after that, they learn digit-based algorithmic addition and subtraction to 1000. A start is made with digit-based algorithmic addition and subtraction with decimal numbers in the context of money.
5	Students add and subtract to 10,000 by mental calculation, also in clever ways and by estimation. Hereafter, this is extended to numbers to 1,000,000, in which students make use of decimally splitting the numbers. A start is made with adding and subtracting bare decimal numbers (3.5 + 0.8; 9.45 – 3.4). Digit-based algorithmic addition and subtraction with whole numbers is done to 10,000 and with decimal numbers in the context of money up to €10,000.
6	Students add and subtract to 1,000,000 by mental calculation, also in clever ways and by estimation. Students add and subtract with decimal numbers (2.55 + 3.5 + 102; 7.85 – 5.4). Digit-based algorithmic addition and subtraction with whole numbers is done to 100,000 and with decimal numbers in the context of money up to €10,000.

PP have digit-based multiplication as a goal for all students, AT has whole-number-based multiplication as a goal for lesser able students, and RR again lets lesser able students choose between whole-number-based or digit-based multiplication.

Finally, differences occur regarding the goals that textbooks set for the end of primary school. For example, the number range within which the students have to solve written multiplication problems differs among the textbooks. The textbook series WiG, AT, and RR have as a goal that students

TABLE 2.4 Sequencing of the Content Related to Addition and Subtraction (Whole Numbers and Decimal Numbers) Over the Grades in the Four Most Widely Used Dutch Textbook Series

Content	Textbook Series			
	WiG	PP	AT	RR
Addition and subtraction facts up to 20	Grades 1–3	Grades 1–3	Grades 1–3	Grades 1–3
Mental addition and subtraction in standard ways	Grades 2–6	Grades 2–6	Grades 2–6	Grades 2–6
Mental addition and subtraction in varying ways	Grades 2–6	Grades 2–6	Grades 2–6	Grades 2–6
Addition and subtraction by estimation	Grades 3–6	Grades 4–6	Grades 2–6	Grades 2–6
Whole-number-based written addition and subtraction	Grade 4	Grades 3–4	Grades 3–5	Grades 4–6
Digit-based algorithmic written addition and subtraction	Grades 4–6	Grades 4–6	Grades 3–6	Grades 4–6

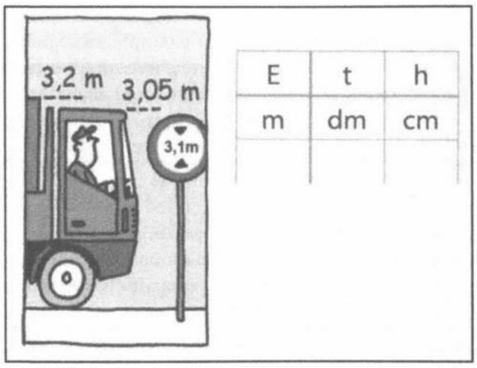


Figure 2.7 A place-value chart in WiG. *Source:* Huitema et al., 2009–2014; students' book Grade 5, p. 8. Reprinted with permission. *Note: E* = eenheden [*U* = units], *t* = tienden [*t* = tenths], *h* = honderdsten [*h* = hundredths]. In Dutch, decimal numbers have a decimal comma instead of a decimal point.

learn to multiply two-digit numbers with three-digit numbers in a digit-based algorithmic way, whereas PP does not go further than multiplying one-digit numbers with three-digit numbers and two-digit numbers with two-digit numbers.

Besides the exercises that are meant for all students, the four textbooks all provide tasks at mostly three levels. For example, WiG distinguishes so-called *one-star, two-stars,* and *three-stars* level tasks. Differences between these levels involve, among other things, the number range used and the complexity of the questioning. Moreover, at the one-star level, more opportunity for repetition is offered and more concrete tasks are given for a longer period of time. For example, in the final lessons in Grade 6 about multiplication by estimation, the one-star level tasks comprise estimation with decimal measurement numbers (Figure 2.8), whereas the two-star level tasks also include estimation with bare decimal numbers (Figure 2.9). The three-star level tasks require more insight, and often provide puzzle-like tasks, such as the task shown in Figure 2.10 (also from the aforementioned lesson), in which students have to use their knowledge of place value in a creative way.

What is lacking in the four textbooks is an overview of the domain-overarching competence of problem solving. This does not mean that the textbook series do not provide assignments that include problem solving.

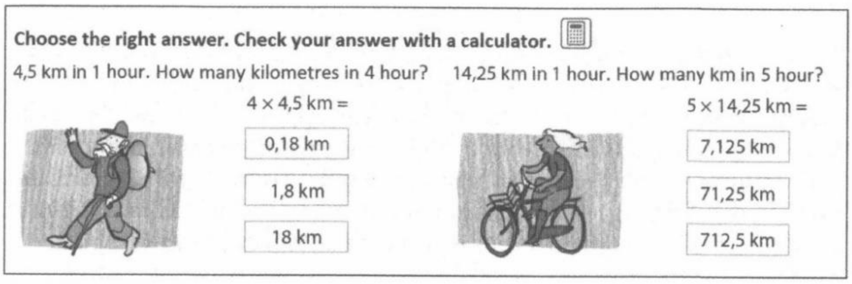


Figure 2.8 A Grade 6 *one-star* level task on multiplication by estimation. *Source:* Huitema et al., 2009–2014, students' book Grade 6, p. 56. Reprinted with permission.

Estimate and choose the right answer.

32 × 1,9 ≈	6,4	64	640	6400
2,9 × 40,3 ≈	1,2	12	120	1200
7 × 349,98 ≈	2,45	24,5	245	2450
98,67 × 30 ≈	20	300	200	3000
3,05 × 5,97 ≈	1,8	18	180	1800

Figure 2.9 A Grade 6 *two-stars* level task on multiplication by estimation. *Source:* Huitema et al., 2009–2014, students' book Grade 6, p. 58. Reprinted with permission.

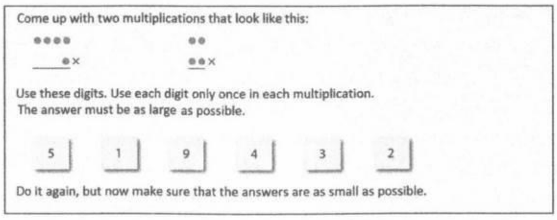


Figure 2.10 A Grade 6 *three-stars* level task on multiplication by estimation. *Source:* Huitema et al., 2009–2014, students' book Grade 6, p. 59. Reprinted with permission.

They do, but only a few assignments are included and not in a systematic way. Furthermore, problem solving tasks are mostly offered in the sections meant for the best students. In contrast, the application of mathematical knowledge and skills in solving straightforward context situations is dealt with in almost every lesson in each textbook series. Regarding another domain-overarching competence, namely using mathematical language, only AT provides an overview of mathematical words per grade.

THE MATHEMATICS CURRICULUM AS REFLECTED IN THE END OF PRIMARY SCHOOL TEST

The compulsory test at the end of primary school serves three purposes. First, the test provides objective information used in making a decision about what level of secondary education a student will attend. Second, the test results are used to know what reference level (1F or 1S) a student has mastered. Third, the test results function for the school inspectorate, next to other indicators, as a measure to assess the quality of a school. So, the end of primary school test can be considered a high-stakes test, both for students and schools.

The test that is developed by Cito and commissioned by the government is called "Centrale Eindtoets" [Central End of Primary School Test]. It is used by a majority of schools in the Netherlands (e.g., Hemker, 2016). Currently, there are several other tests developed by commercial testing companies that are approved by the government. The criteria for approval, which are also the criteria for the Central End of Primary School Test, are described in the "Toetswijzer Eindtoets PO" [Directions for End of Primary School Tests] (CvTE, 2014).

Directions for the Mathematics End of Primary School Tests

End of primary school tests must meet a number of demands with respect to validity, reliability, and content. Concerning the content, to which we confine ourselves here, end of primary school tests must cover levels 1F and 1S for all domains included in the Reference Framework (number, rational numbers and ratios, measurement and geometry, and data handling). For each domain, a minimum and maximum proportion of test items is prescribed. Also, the competencies (using mathematical language, making connections between procedures and concepts, and carrying out applications in context situations and bare number problems) named in the Reference Framework must be dealt with in an end of primary school test. The same applies to the performance expectations (knowing by heart, being able to use, and understanding).

There are also three additional specific demands. The first is that a test must contain both context problems and bare number problems, with a minimum proportion of thirty and twenty percent of all items, respectively. This demand is a direct outcome of a debate about whether mathematics education at primary school should include context situations or focus on bare number calculation. The second demand is that end of primary school tests should allow the use of scrap paper in at least eighty percent of all items, adhering to research that indicates using scrap paper was of more influence on getting a correct answer than use of a particular calculation procedure (Hickendorff, 2011). The last demand is that a test should measure whether students are able to use a calculator in a reasonable way.

The Central End of Primary School Test for Mathematics

Because a majority of schools use the Central End of Primary School Test (hereafter called the "Central Test"), we limit ourselves here to this test. The Central Test covers all the domains of the Reference Framework (Table 2.5), but not all performance expectations mentioned in the Reference Framework. This test does not (yet) contain test items assessing the ability to use a calculator, partly because this would require too many test items (CvTE, 2015a). Furthermore, the ability to make use of measurement devices is not assessed, due to the fact that the Central Test used now has a multiple-choice format.

For the Central Test, the Directions for End of Primary School Tests are extended with detailed specifications regarding the content and

TABLE 2.5 Content Included in the Central End of Primary School Test for Mathematics

Domain Mentioned in Reference Framework	Content Included in Central End of Primary School Test
Numbers	• Number sense • Operations with whole numbers and decimal numbers • Operations with fractions
Ratios	• Identifying ratios and expressing them as part-whole, fractions, percentages • Solving problems with ratios (e.g., recipes) • . . .
Measurement and Geometry	• Measurement: length and circumference, area, volume, weight, time and speed, money • Geometry: shapes and figures, orientation and localization, symmetry and patterns
Data Handling	• Tables • Graphs

Note: From CvTE (2015a).

performance expectations. For example, for basic operations with whole numbers and decimal numbers, these include the following (CvTE, 2015a, pp. 53, 55):

- adding and subtracting using properties of numbers and operations, including calculation with numbers with zeroes (e.g., 4000 + 60,000; 180,000 – 2,000);
- using standard procedures for addition and subtraction with large whole numbers and decimal numbers with multiple digits;
- adding and subtracting by estimation with large whole numbers and with decimal numbers (49.95 + 128.95 + 32.35 is about 50 + 130 + 30);
- multiplying and dividing by using properties of numbers and operations, including multiplying and dividing whole numbers and decimal numbers by 10, 100, 1000 (1.8 × 100), and multiplying and dividing whole numbers by other numbers with zeroes (60 × 400; 3200 ÷ 40);
- using standard procedures for multiplication and division with large whole numbers and decimal numbers;
- interpreting the remainder of a division problem (e.g., transporting 659 children in buses; each bus can transport 45 children; 659 ÷ 45 = 14 remainder 29, so there are 15 buses needed); and
- multiplying and dividing by estimation with large whole numbers and decimal numbers (49 × 198.97 is about 50 × 200).

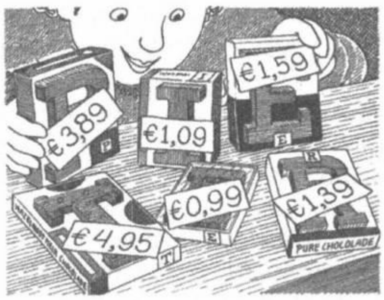
Because of the amount of content included in the Central Test, for language and mathematics together, it takes three mornings, including breaks, to administer the test. The 2015 version of the Central Test included 85 items for mathematics. In all items, the use of scrap paper was allowed. Figure 2.11 shows four items of the 2015 Central Test.

THE COHERENCE OF THE MATHEMATICS CURRICULUM

The coherence of a curriculum is of decisive influence on students' opportunities to learn (Schmidt, Houang, & Cogan, 2002). Curricular coherence can be considered in different ways, of which the alignment of different curriculum resources, referring to the degree in which resources agree with one another, can be seen as one of the most elementary forms (Schmidt, Wang, & McKnight, 2005). We use the term in this way, which is visualized in the Dutch *curricular spider web model* (Van den Akker, 2003; see Figure 2.12). This model illustrates the coherence of the several elements of a curriculum, but at the same time it also makes clear how vulnerable

Rob picked strawberries for two weeks.
He earned 199 euro per week. From his payment he bought a telephone of 99 euro.
How much money has he got left?

A € 297,- C € 299,-
B € 298,- D € 301,-



Pieter buys these 6 chocolate letters.
About how much euro does he have to pay?

A 11 euro C 13 euro
B 12 euro D 14 euro

1 000 001 – 888 888 =

A 111113 C 121213
B 111123 D 222223

88 x 99 =

A 1584 C 8712
B 7272 D 8799

Figure 2.11 Four items on basic operations from the 2015 Central Test. *Source:* CvTE, 2015b. Reprinted with permission.

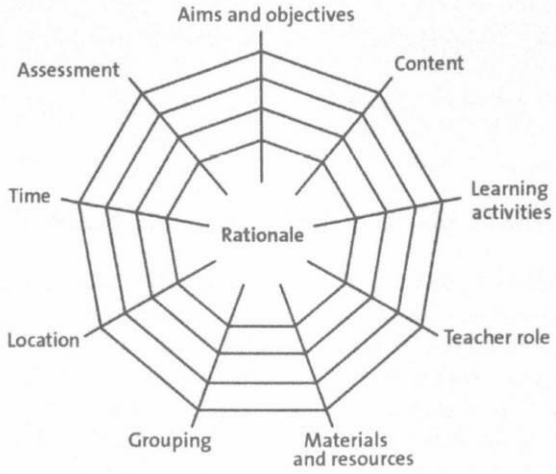


Figure 2.12 The curricular spider web. *Source:* Van den Akker, 2003.

a curriculum is. When it is pulled too hard at the ends, the spider web can break. For example, if learning materials do not fit the content to be learned, then learning goals probably will not be achieved.

The situation in which decisions regarding the curriculum are made by different actors—a government, textbook publishers, testing organizations—who each may have their own goals and visions, can be considered a threat to curricular coherence (Schmidt, Wang, & McKnight, 2005). This situation specifically applies to the Netherlands with its policy of freedom of education. Therefore, in this section, we address whether the documents that describe the intended curriculum (the Core Goals and the Reference Framework) are in alignment with each other, and whether the documents that we consider as the potentially implemented curriculum (the TAL teaching-learning trajectories, the textbook series, and the end of primary school test) correspond with the intended curriculum.

Coherence Within the Intended Curriculum

The Core Goals document and the Reference Framework give descriptions of the same content and performance expectations. All domains, content, and performance expectations included in the Core Goals document are also mentioned in the Reference Framework. The same goes for the overarching competencies of using mathematical language and problem solving, although the latter has a less prominent place in the Reference Framework than in the Core Goals document.

As noted earlier, the Reference Framework is elaborated in more detail than the Core Goals document and the Reference Framework distinguishes two levels in the attainment targets for the end of primary school. There are two other significant differences between the two documents. The first is that the Core Goals document indicates "what primary schools should be *aiming for* regarding the development of their students" (Ministry of Education, 2006, p. 1), whereas the Reference Framework describes "what students should know and be able to do regarding Dutch language and mathematics" (Ministry of Education, 2009, p. 5). Because of the latter, in 2015 the end of primary school test became mandatory, which was not previously the case. Second, although the Reference Framework contains the same content and performance expectations as the Core Goals document (CvTE, 2014), several overarching competencies emphasized in the Core Goals document are not included in the Reference Framework. This is, for example, the case for asking mathematical questions, using mathematical literacy, and giving and receiving criticism as a mathematical method. Furthermore, issues regarding attitudes mentioned in the Core Goals, such as feeling challenged and doing mathematics with satisfaction and pleasure, are also not referred to in the Reference Framework. Thus, compared to the Core Goals document, albeit the Reference Framework is more detailed in its descriptions, it is more limited with respect to mathematical attitude and overarching competence foci.

Coherence Within the Potentially Implemented Curriculum

The TAL teaching-learning trajectories, which were developed between 1996 and 2007, are based on the 1993/1998 version of the Core Goals document. Because the 2006 Core Goals document (Figure 2.3) is far more global than the 1993/1998 version was, it was expected "that the TAL teaching-learning trajectories and the included intermediate attainment targets, will play a large role in guiding decisions about mathematical content" (Van den Heuvel-Panhuizen & Wijers, 2005, p. 294). Currently, indeed, in all four most frequently used textbooks series it is explicitly stated in the accompanying teacher guidelines that the textbooks are based—next to the Core Goals and the Reference Framework—on the TAL teaching-learning trajectories (Bazen et al., 2009–2013, teacher guidelines, p. 4;[5] Huitema et al., 2009–2014, teacher guidelines, p. 2; Van Beusekom et al., 2009–2013, teacher guidelines, p. 5; Van den Bosch-Ploegh et al., 2009–2013, teacher guidelines, p. 12, p. 14). That this indeed is the case is evidenced by the corresponding ways in which content and performance expectations are aligned in the textbook series with the TAL trajectories. This is also true for the use of certain learning

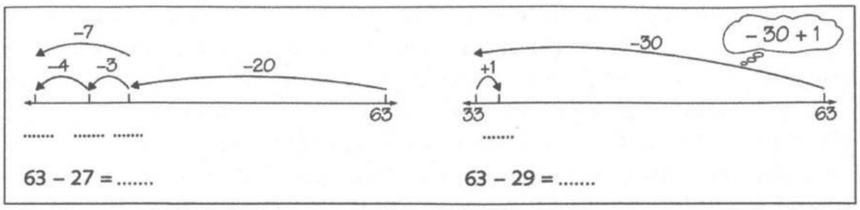


Figure 2.13 Use of the empty number line in RR. *Source:* Bazen et al., 2009–2013; students' book Grade 2, p. 45. Reprinted with permission.

facilitators as suggested by TAL, such as the empty number line, which is present in all four textbooks series (see Figure 2.13 for an example).

Despite the fact that all four textbook series have a connection with TAL, there are several differences in their elaborations of content and performance expectations (some of which were discussed in the section about textbooks) and the provision of learning facilitators, such as models. Furthermore, not everything emphasized in TAL is also present in all four textbook series. We discuss more about this in the following section.

Coherence Between the Intended and the Potentially Implemented Curriculum

The documents of the potentially implemented curriculum—the TAL teaching-learning trajectories, the four most frequently used textbook series, and the Central Test—include all the domains prescribed in the intended curriculum. They all comprise numbers and operations, ratios, measurement, geometry, and data handling. With respect to the four textbook series, analyses carried out by SLO (2012a, 2012b, 2012c, 2012d) have established that the textbooks meet the standards as described in the Core Goals document. However, it should be noted that these analyses were done very broadly and the Reference Framework was not (yet) included in these analyses.

Although the intended curriculum documents are global in nature, the potentially implemented curriculum documents provide detailed elaborations of content and performance expectations. As an example of the similarities and differences that currently exist among the curriculum documents, Table 2.6 contains a list of the ways in which written multiplication is dealt with in the Core Goals document, the Reference Framework, the TAL teaching-learning trajectories, the four textbook series, and the Central Test.

The Core Goals document and the Reference Framework prescribe that students should learn a form of written multiplication, but do not indicate what specific form (algorithmic digit-based or whole-number-based) that

TABLE 2.6 Similarities and Differences Among the Curriculum Documents for Written Multiplication

Curriculum Document	How This Document Deals With Written Multiplication
Core Goals	• Students learn written multiplication in more or less curtailed standardized ways.
Reference Framework (level 1F and 1S)	• Multiplication of a one-digit number with a two-digit or three-digit number. • Multiplication of a two-digit number with a two-digit number.
TAL Teaching-Learning Trajectory	• The most curtailed digit-based algorithmic multiplication is not considered an attainment target for lesser able students.
Textbook Series	• WiG, AT, and RR have as a goal that students learn to multiply two-digit numbers with three-digit numbers. Digit-based algorithmic multiplication is derived from whole-number-based written multiplication. • PP has as a goal that students learn to multiply one-digit numbers with three-digit numbers and two-digit numbers with two-digit numbers. No relationship is made between whole-number-based and digit-based multiplication. • WiG and PP have digit-based multiplication as a goal for all students. AT has whole-number-based multiplication as a goal for lesser able students. RR lets lesser able students choose between whole-number-based or digit-based multiplication.
Central Test	• Using standard procedures for multiplication with large whole numbers and decimal numbers.

should be. Also, the Directions for the End of Primary School Tests document do not prescribe which multiplication form should be used. The same goes for the Central Test. TAL, however, does provide an indication of the form that students should learn: the most curtailed form of digit-based multiplication is not considered an attainment target for lesser able students. The approach to written multiplication in the four textbook series varies. In agreement with TAL, in AT and RR, digit-based multiplication is not considered an attainment target for lesser able students. In WiG and PP, however, digit-based multiplication is an attainment target for all students, including the less able ones. Another difference between the textbook series is the attainment target regarding the number range in which students should be able to work. The textbook series PP has as a goal that students learn to multiply one-digit numbers with three-digit numbers and two-digit numbers with two-digit numbers, which precisely corresponds with the number range prescribed in the Reference Framework. The other textbook series aim for all students learning to multiply two-digit numbers with three-digit numbers. Finally, in WiG, AT, and RR, digit-based multiplication is derived from whole-number-based multiplication, whereas PP does not make a connection between the two forms.

Regarding the coherence between the intended curriculum and the end of primary school tests, we must say, there is a weak point. According to the Directions for the End of Primary School Tests, these tests have to "test students on their knowledge and skills regarding the Reference Framework" (CvTE, 2014, p. 17); the same document also indicates that this automatically means that the content and performance expectations of the Core Goals document are covered (CvTE, 2014). However, the latter is not necessarily true, because some overarching competencies included in the Core Goals are missing in the Reference Framework. Furthermore, some performance expectations (such as being able to use a calculator and measuring devices) are not included (yet) in the Central Test.

FINAL REMARKS

As discussed earlier, freedom of education in the Netherlands implies that there are few restrictions in developing textbooks, and that schools may choose whatever textbook series they want to use. However, because different textbooks may provide different opportunities to learn (Van Zanten & Van den Heuvel-Panhuizen, 2014) and because textbooks have a determining role for daily teaching practice in the Netherlands, we conclude this chapter with some remarks considering textbook series.

The examples provided in this chapter suggest that different elaborations within the four most frequently used textbook series fall within the boundaries of the globally described intended curriculum. However, we raise two issues.

The first issue is about the differentiated attainment targets as provided by the Reference Framework in which the levels 1F and 1S are distinguished. All four textbook series have incorporated these levels by including differentiated tasks. For example, the learning route following the *one-star* tasks in WiG is supposed to lead to mastery of the 1F level, and the route of the *two-stars* tasks should lead to the mastery of the 1S level. However, whether such differentiated learning routes within textbooks indeed lead to the mastery of the levels aimed at is not known. The fact that currently only about 45% of students at the end of primary school master the 1S level (Educational Inspectorate, 2016[6]), which is meant for a majority of the students, raises the question of whether the 1S level is well enough incorporated in the textbooks, and also how teachers deal with the differentiated routes provided by the textbooks.

The second issue concerns the domain overarching competencies, especially problem solving. Although problem solving is mentioned in both the Core Goals document and the Reference Framework, and the TAL teaching-learning trajectories explicitly emphasize the importance of it, there is only limited attention on problem solving in the four textbook series, and mainly only for the best students. This means that most students have only few opportunities to develop this mathematical competence.

Both issues—having a structure in the textbooks that clearly leads to the 1S level and offering students the opportunity to develop problem solving competencies—are definitely tasks for textbook developers to address, but to improve textbook series at this point requires that all curriculum levels be involved. Only then can the coherence of the curriculum be secured and the curriculum fulfill its role as a steering tool for high quality education.

ACKNOWLEDGMENTS

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NOTES

1. Successively, these TAL trajectories have also been published in English (Gravemeijer et al., 2016; Van den Heuvel-Panhuizen, 2008; Van den Heuvel-Panhuizen & Buys, 2008; Van Galen et al., 2008).
2. (See http://TULE.slo.nl/). TULE stands for "Tussendoelen en leerlijnen" [Intermediate goals and teaching-learning trajectories]. Two of the three authors of TULE mathematics (Buijs, Klep, & Noteboom, 2008) were also involved in the development of TAL.
3. In the TAL teaching-learning trajectory (see Van den Heuvel-Panhuizen, 2008), this whole-number-based calculation is called *column calculation*.
4. In the TAL teaching-learning trajectory (see Van den Heuvel-Panhuizen, 2008), this digit-based algorithmic calculation is called *algorithmic calculation*.
5. In the RR guidelines, it only says "teaching-learning trajectories," but one of the authors of this textbook series confirmed that here the TAL teaching-learning trajectories are meant.
6. The same study shows that 90% of students master the 1F level at the end of primary school.

REFERENCES

Bakker, N., Noordman, J., & Rietveld-van Wingerden, M. (2010). *Vijf eeuwen opvoeden in Nederland* [Five centuries of education in the Netherlands]. (2nd Edition). Assen, the Netherlands: Van Gorcum.

Bazen, K., Bokhove, J., Borghouts, C., Buter, A., Kuipers, K., & Veltman, A. (2009–2013). *Rekenrijk 3e editie* [Kingdom of arithmetic / Rich arithmetic]. Groningen/Houten, the Netherlands: Noordhoff Uitgevers.

Boekholt, P., & De Booy, E. (1987). *Geschiedenis van de school in Nederland.* [A history of schools in the Netherlands]. Assen, the Netherlands: Van Gorcum.

Buijs, K., Klep, J., & Noteboom, A. (2008). *TULE rekenen/wiskunde. Inhouden en activiteiten bij de kerndoelen van 2006.* [TULE Mathematics. Content and activities for the 2006 Core Goals]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

Committee Parliamentary Research Education. (2008). *Tijd voor onderwijs.* [Time for education]. Den Haag, the Netherlands: Sdu Uitgevers.

CvTE. (2014). *Toetswijzer eindtoets PO.* [Directions for end of primary school tests]. Utrecht, the Netherlands: College voor Toetsen en Examens.

CvTE. (2015a). *Toetswijzer bij de Centrale eindtoets PO taal en rekenen.* [Directions for the Central End of primary school test for language and mathematics]. Utrecht, the Netherlands: College voor Toetsen en Examens.

CvTE. (2015b). *Central Test 2015.* Utrecht, the Netherlands: College voor Toetsen en Examens.

De Wit, C. (1997). *Over tussendoelen gesproken. Tussendoelen als component van leerlijnen.* [Talking about intermediate goals. Intermediate goals as a component of teaching-learning trajectories]. 's-Hertogenbosch, the Netherlands: KPC Onderwijs Innovatie Centrum.

Education Council. (2012). *Artikel 23 Grondwet in maatschappelijk perspectief [Article 23 of the Constitution in societal perspective].* Den Haag, the Netherlands: Education Council of the Netherlands.

Educational Inspectorate. (2016). *Taal en rekenen aan het einde van het basisonderwijs.* [Language and mathematics at the end of primary education]. Utrecht, the Netherlands: Educational Inspectorates.

Expertgroep Doorlopende Leerlijnen. (2008). *Over de drempels met taal en rekenen.* [Crossing the thresholds with language and mathematics]. Enschede, the Netherlands: SLO.

Gardner, M. (1985). *The magic numbers of Dr. Matrix.* New York, NY: Prometeus Books.

Goodlad, J. (Ed.). (1979). *Curriculum inquiry.* New York, NY: McGraw-Hill.

Gravemeijer, K., Figueiredo, N., Feijs, E., Van Galen, F., Keijzer, R., & Munk, F. (2007). *Meten en meetkunde in de bovenbouw. Tussendoelen annex leerlijnen bovenbouw basisschool.* [Measurement and geometry in the upper grades. Teaching-learning trajectory for the upper grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Gravemeijer, K., Figueiredo, N., Feijs, E., Van Galen, F., Keijzer, R., & Munk, F. (2016). *Measurement and geometry in the upper primary school.* Rotterdam, the Netherlands/Tapei, Taiwan: Sense.

Hemker, B. (2016). *Peiling van de rekenvaardigheid, de taalvaardigheid en wereldoriëntatievaardigheden in jaargroep 8 van het basisonderwijs in 2015.* [Survey on skills in mathematical, language and "world orientation" in Grade 6 in 2015]. Arnhem, the Netherlands: Cito.

Hickendorff, M. (2011). *Explanatory latent variable modeling of mathematical ability in primary school.* Leiden, the Netherlands: Leiden University (diss.).

Hop, M. (Ed.). (2012). *Balans van het reken-wiskundeonderwijs halverwege de basisschool. Periodieke peiling van het onderwijsniveau 5.* [Balance of mathematics education halfway primary school. Periodic assessment of the education level 5]. Arnhem, the Netherlands: Cito.

Huitema, S., Erich, L., Van Hijum, R., Nillesen, C., Osinga, H., Veltman, H., & Van de Wetering, M. (2009–2014). *De Wereld in Getallen.* 's-Hertogenbosch, the Netherlands: Malmberg.

Letschert, J. (1998). *Wieden in een geheime tuin. Een studie naar kerndoelen in het Nederlandse basisonderwijs.* [Weeding in a secret garden. A study of core goals in Dutch primary education]. Enschede, the Netherlands: SLO (Netherlands Institute for Curriculum Development.

Meelissen, M. R. M., Netten, A., Drent, M., Punter, R.A., Droop, M., & Verhoeven, L. (2012). *PIRLS en TIMSS 2011. Trends in leerprestaties in Lezen, Rekenen en Natuuronderwijs.* [PIRLS and TIMSS 2011. Trends in achievement in Reading, Mathematics and Science.] Nijmegen / Enschede: Radboud University / Twente University.

Ministry of Education. (1993/1998). *Kerndoelen basisonderwijs.* [Core Goals Primary Education]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2006). *Kerndoelen basisonderwijs.* [Core Goals Primary Education]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2007). Reken- en taalvaardigheid en doorlopende leerlijnen. Brief aan de Tweede Kamer. [Mathematical and language skills and continual learning trajectories—Letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2008). Beleidsreactie "Tijd voor onderwijs"—Brief aan de Tweede Kamer [Reaction to "Time for education"—Letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2009). *Referentiekader taal en rekenen* [Reference Standards language and mathematics]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2013). Artikel 23 Grondwet in maatschappelijk perspectief—Brief aan de Tweede Kamer [Article 23 of the Constitution in a societal perspective—Letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2015). Meer ruimte voor nieuwe scholen: Naar een moderne interpretatie van artikel 23—Brief aan de Tweede Kamer [More room for new schools: Towards a modern interpretation of article 23—Letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Scheltens, F., Hemker, B., & Vermeulen, J. (2013). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 5* [Balance of mathematics education at the end of primary school]. Arnhem, the Netherlands: Cito.

Schmidt, W., Houang, R., & Cogan, L. (2002). A coherent curriculum. The case of mathematics. *American Educator, 26(2),* 1–17.

Schmidt, W., Wang, H., & McKnight, C. (2005). Curriculum coherence: An examination of U.S. mathematics and science content standards from an international perspective. *Journal of Curriculum Studies, 35,* 525–559.

SLO. (2012a). *Kerndoelenanalyse Alles Telt* [Core goal analysis Alles Telt]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

SLO. (2012b). *Kerndoelenanalyse De Wereld in Getallen* [Core goal analysis De Wereld in Getallen]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

SLO. (2012c). *Kerndoelenanalyse Pluspunt* [Core goal analysis Pluspunt]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

SLO. (2012d). *Kerndoelenanalyse Rekenrijk* [Core goal analysis Rekenrijk]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

Thijs, A., & Van den Akker, J. (2009). *Curriculum in development.* Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

Treffers, A., Van den Heuvel-Panhuizen, M., & Buys, K. (1999). *Jonge kinderen leren rekenen. Tussendoelen Annex Leerlijnen Hele Getallen Onderbouw Basisschool* [Young children learn mathematics. A learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers for the lower grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Valverde, G., Bianchi, L., Wolfe, R., Schmidt, W., & Houang, R. (2002). *According to the Book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks.* Dordrecht, the Netherlands: Kluwer Academic.

Van Beusekom, N., Fourdraine, A., & Van Gool, A. (Eds.). (2009–2013). *Pluspunt.* 's-Hertogenbosch, the Netherlands: Malmberg.

Van den Akker, J. (2003). Curriculum perspectives: An introduction. In J. Van den Akker, W. Kuiper, & U. Hameyer (Eds.), *Curriculum landscapes and trends.* (pp. 1–10). Dordrecht, the Netherlands: Kluwer Academic.

Van den Bosch-Ploegh, E., Van den Brom-Snijders, P., Hessing, S., Van Kraanen, H., Krol, B., Nijs-van Noort, J., . . . , Vuurmans, A. (2009–2013). *Alles telt 2e editie* [Everything counts]. Amersfoort, the Netherlands: ThiemeMeulenhoff.

Van den Heuvel-Panhuizen, M. (Ed.). (2008). *Children learn mathematics. A learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers in primary school.* Rotterdam, the Netherlands/Tapei, Taiwan: Sense.

Van den Heuvel-Panhuizen, M., & Buys, K. (Eds.). (2004). *Jonge kinderen leren meten en meetkunde. Tussendoelen annex leerlijnen onderbouw basisschool* [Young children learn measurement and geometry. Teaching-learning trajectory for the lower grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Van den Heuvel-Panhuizen, M., & Buys, K. (Eds.). (2008). *Young children learn measurement and geometry. A learning-teaching trajectory with intermediate attainment targets for the lower grades in primary school.* Rotterdam, the Netherlands/Tapei, Taiwan: Sense.

Van den Heuvel-Panhuizen, M., Buys, K., & Treffers, A. (Eds.). (2001). *Kinderen leren rekenen. Tussendoelen annex leerlijnen. Hele getallen bovenbouw basisschool* [Children learn mathematics. A teaching-learning trajectory for whole numbers in the upper grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Van den Heuvel-Panhuizen, M. & Wijers, M. (2005). Mathematics standards and curricula in the Netherlands. *ZDM—The International Journal on Mathematics Education, 37*(4), 287–307.

Van Galen, F., Feijs, E., Figueiredo, N., Gravemeijer, K., Van Herpen, E., & Keijzer, R. (2005). *Breuken, procenten, kommagetallen en verhoudingen. Tussendoelen annex leerlijnen bovenbouw basisschool* [Fractions, percentages, decimal numbers and ratio. Teaching-learning trajectory for the upper grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Van Galen, F., Feijs, E., Figueiredo, N., Gravemeijer, K., Van Herpen, E., & Keijzer, R. (2008). *Fractions, percentages, decimal numbers and ratio. A learning-teaching trajectory for Grade 4, 5 and 6.* Rotterdam, the Netherlands/Tapei, Taiwan: Sense.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2014). Freedom of design: the multiple faces of subtraction in Dutch primary school textbooks. In Y. Li & G. Lappan (Eds.), *Mathematics curriculum in school education.* (pp. 231–259). Dordrecht, the Netherlands: Springer.

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